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Nonlinear analysis of infinite beams on granular bed-stone column-reinforced earth beds under moving loads

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Abstract

The present paper pertains to the modeling and the analysis of an infinite beam subjected to a concentrated load moving at a constant velocity and resting on granular bed-stone column-reinforced earth beds. The granular fill has been modeled as a Pasternak shear layer, while the naturally occurring saturated soft soil has been idealized by a Kelvin–Voigt model and the stone columns by Winkler springs. The nonlinear behavior of the granular fill, the stone columns and the soft soil has been represented by hyperbolic constitutive relationships. The governing differential equations of the soil–foundation system have been derived and presented in a non-dimensional form. These equations have been solved using appropriate boundary conditions by means of an iterative Gauss–Siedel technique. A detailed parametric study has been conducted to investigate the influence of various parameters, such as the magnitude and the velocity of the applied load, viscous damping, the diameter and the spacing of the stone columns, the ultimate resistance of the soft soil and the stone columns, the relative stiffness of the stone columns and the average degree of consolidation on the response of the soil–foundation system. All these parameters have been found to significantly influence the response of the infinite beam. However, the ultimate shear resistance of the granular fill has been found to have a negligible effect on the response of the system.

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Keywords: Infinite beams; Nonlinear behavior; Stone columns; Viscous damping; Ground improvement

1. Introduction

Nowadays, numerous ground improvement techniques are adopted for all kinds of infrastructural development on poor/soft soils, especially in coastal areas. The installment of stone columns is one of these techniques. A stone column is constructed by filling a cylindrical cavity with

granular material. Soil improvement via the stone columns is achieved by accelerating the consolidation of the soft soil due to the shortened drainage path, by an increase in the load-carrying capacity and/or by a decrease in the settlement due to the inclusion of stronger granular material. Apart from improving the ground below the foundations of residential as well as industrial buildings, stone columns are also installed in soft soils or loose sand for railroads and roadways due to the stringent settlement restrictions and the fast-track nature of the projects (Arulrajah et al., 2009).

Various studies have been conducted to investigate the behavior of several types of foundations on stone-column treated grounds. Some of these include Balaam and Booker (1981), Schweiger and Pande (1986), Canetta and Nova (1989), Alamgir et al. (1996), Poorooshasb and Meyerhof (1997), Lee and Pande (1998), Shahu et al. (2000), Elshazly et al. (2007), Deb et al. (2007), Deb (2008), Maheshwari and Khatri (2010, 2011), etc. In all of these

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studies, various types of foundations have been analyzed for static load conditions only.

In addition, many studies are available on the analysis of rails, treated as infinite beams, under the action of moving loads. Various researchers in the past have studied the response of rails as infinite beams on elastic foundations, subjected to concentrated moving loads as well as dynamic loads, using different numerical techniques (Kenny, 1954; Kerr, 1974; Saito and Teresawa, 1980; Wang et al., 1984; Duffy, 1990, etc.). In the recent past, Jaiswal and Iyenger (1997) presented a dynamic analysis of a railway track under moving vibratory masses and concluded that the mass of moving loads significantly reduces the resonant frequency and the critical velocity. Momoya et al. (2005) developed a relevant performance-based design method for railway asphalt roadbeds, in which the resilient and the residual deformation characteristics of railway roadbeds and subgrade were investigated by means of scale model tests. Mallik et al. (2006) studied the response of an infinite beam resting on one parameter, as well as two-parameter lumped models, subjected to a moving load and considering both damped and undamped cases. In all of these studies, no ground improvement technique was adopted, and therefore, they may not be applicable to poor/soft soils. In view of this, Maheshwari et al. (2004, 2006) proposed simplified linear models for the analysis of infinite beams on geosynthetic-reinforced earth beds under moving loads; the influence of viscous damping was not considered in the analysis. In these models, ground improvement was performed by means of only a geosynthetic layer, which may not be very effective in reducing the settlement and in enhancing the bearing capacity.

From a critical review of the literature, it is clear that no study is available on the analysis of infinite beams on stone column-treated grounds, and therefore, a nonlinear model has been introduced in the present work for the analysis of infinite beams under concentrated loads moving at a constant velocity and resting on a granular fill-stone column-reinforced soft soil system. The overall behavior has been investigated through a detailed parametric study. The numerical solution of the proposed model has been

obtained by an iterative finite difference method and all the results have been presented in a non-dimensional form.

2. Soil–foundation system under consideration and proposed model

Fig. 1 shows the longitudinal section of a rail idealized as an infinite beam resting on a ballast layer idealized as a granular fill-stone column-reinforced soft soil system. The beam is founded on a granular fill layer of thickness H overlying saturated soft soil. The shear modulus of the granular fill layer is G . The diameter and the spacing of the stone columns are d and s , respectively.

The conceptual idealization of the physical model for the soil–foundation system is presented in Fig. 2. The soft soil subgrade has been idealized by the Kelvin–Voigt model and the stone columns as Winkler springs. The granular fill layer has been modeled as a Pasternak shear layer. A spatial domain of length $2L$ has been considered in the analysis, large enough for the beam to be considered as an infinite beam. The hyperbolic nonlinear stress–displacement relationship proposed by Kondner and Zelasko (1963) has been considered to exhibit the behavior of the granular fill and the stone columns. The stress–displacement response of the saturated soft soil has been represented by the hyperbolic relationship proposed by Kondner (1963). The stone columns were assumed to have been installed throughout the depth of the natural soil bed overlying the rigid stratum. The influence of the disturbance to the soil during the installation of the stone columns has been disregarded in the analysis.

3. Analysis

A free body diagram for the granular fill layer (idealized as a Pasternak shear layer) has been considered, and the vertical force equilibrium equation for this granular fill layer can be written as

$$q = q_s - G H \frac{d^2 w}{dx^2} \quad (1)$$

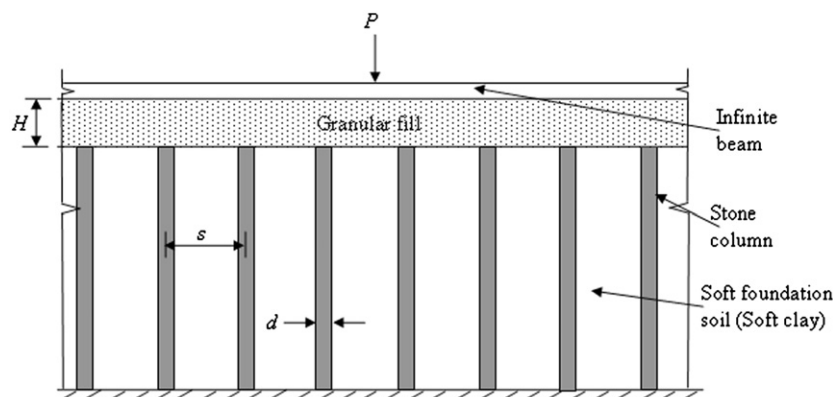


Fig. 1. Definition sketch of rail–geosynthetic reinforced granular fill–poor soil system.

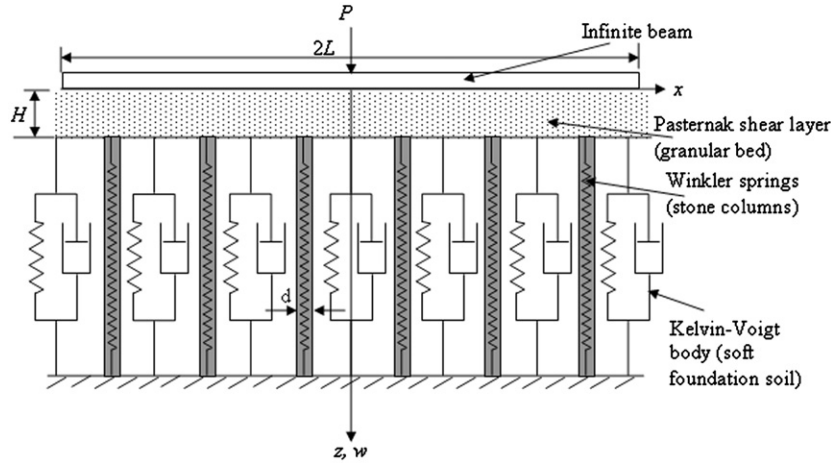


Fig. 2. Definition sketch of proposed model.

where q is the reaction of the granular fill on the beam, q_s is the vertical force interaction between the granular shear layer and the saturated soft foundation soil, G and H are the shear modulus and the thickness of the granular fill layer, respectively, w is the vertical deflection and x is the coordinate along the length of the foundation beam.

The shear modulus of the granular layer can be expressed by considering the hyperbolic shear stress–shear strain response (Ghosh and Madhav, 1994) as

$$G = \frac{G_o}{[1 + (G_o |dw/dx|)/\tau_u]^2} \quad (2)$$

where G_o is the initial shear modulus of the shear layer and τ_u is the ultimate shear resistance of the granular layer.

The vertical force interaction between the granular shear layer and the saturated soft foundation soil, q_s , at any time $t > 0$, can be expressed by employing the effective stress principle as

$$q_s = \bar{\sigma} + u_e \quad (3)$$

where $\bar{\sigma}$ and u_e are the average effective stress and the average excess pore water pressure, respectively, at time t in the spring dashpot system.

Considering the hyperbolic nonlinear stress–displacement relationship (Kondner, 1963), $\bar{\sigma}$ can be expressed as

$$\bar{\sigma} = \frac{k_{so} w}{1 + k_{so}(w/q_u)} \quad (4)$$

where k_{so} is the initial modulus of the subgrade reaction and q_u is the ultimate bearing capacity of the saturated soft soil.

Combining Eqs. (3) and (4), one gets

$$q_s = \frac{k_{so} w}{1 + k_{so}(w/q_u)} + u_e \quad (5)$$

The average excess pore water pressure at any time t can be expressed as follows:

$$u_e = u_o(1 - U) \quad (6)$$

where u_o is the initial pore water pressure and U is the average degree of consolidation at time t , which is due to vertical (U_v)

as well as radial drainage (U_r) and is expressed as

$$U = 1 - (1 - U_r)(1 - U_v) \quad (7)$$

Combining Eqs. (5) and (6),

$$q_s = \frac{k_{so} w}{1 + k_{so}(w/q_u)} + u_o(1 - U) \quad (8)$$

Initially, i.e., at time $t = 0$, stress exists at the interface of the granular fill and the saturated soft soil is carried by the excess pore water pressure within the surrounding soil. In view of this, Eq. (8) can be rewritten as

$$q_s = \frac{k_{so} w}{1 + k_{so}(w/q_u)} + q_s(1 - U) \quad (9)$$

or

$$q_s = \frac{k_{so} w}{U[1 + k_{so}(w/q_u)]} \quad (10)$$

The vertical force interaction between the granular shear layer and the saturated soft foundation soil, q_c , can be written as (Kondner and Zelasko, 1963)

$$q_c = \frac{k_{co} w}{1 + k_{co}(w/q_{cu})} \quad (11)$$

where k_{co} and q_{cu} are the initial modulus of the subgrade reaction and the ultimate bearing capacity of the stone columns, respectively.

The reaction of the granular fill on the beam can therefore be written as

$$q = \frac{k_{so} w}{U[1 + k_{so}(w/q_u)]} - GH \frac{d^2 w}{dx^2} \quad \text{within the saturated soft soil region} \quad (12a)$$

and

$$q = \frac{k_{co} w}{1 + k_{co}(w/q_{cu})} - GH \frac{d^2 w}{dx^2} \quad \text{within the stone column region} \quad (12b)$$

The differential equation of an infinite beam with a moving load can be obtained by considering the bending of an elemental segment. The differential equation of the

beam with the uniform cross section can be written as follows:

$$EI \frac{\partial^4 w}{\partial x^4} + \rho \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + q = P(x, t) \quad (13)$$

where EI is the flexural rigidity of the infinite beam, ρ is the mass per unit length of the beam, c is the coefficient of viscous damping per unit length of the beam and $P(x, t)$ is the applied load intensity.

Eqs. (12) and (13) govern the response of the proposed model. For particular values of parameters, these equations govern the response of the existing models for infinite beams on elastic foundations subjected to moving loads (Kenny, 1954; Mallik et al., 2006).

4. Solution of governing differential equations

For simplicity, a distance ξ from the point of action of load at time t has been considered as $\xi = x - vt$, where v is the constant velocity at which the load is moving on the infinite beam. Eqs. (12) and (13) can be written as

$$q = \frac{k_{so} w}{U[1 + k_{so}(w/q_u)]} - GH \frac{d^2 w}{d\xi^2} \quad (14a)$$

within the saturated soft soil region

and

$$q = \frac{k_{co} w}{1 + k_{co}(w/q_{cu})} - GH \frac{d^2 w}{d\xi^2} \quad (14b)$$

within the stone column region

$$EI \frac{d^4 w}{d\xi^4} + \rho v^2 \frac{d^2 w}{d\xi^2} - c \frac{dw}{d\xi} + q = P(\xi) \quad (15)$$

To observe the settlement response of the proposed model, Eqs. (14) and (15) have been written in a non-dimensional form employing the following non-dimensional parameters:

$\xi^* = \xi/L$, $W = w/L$, $G^* = G H/k_{so} L^2$, $G_o^* = G_o H/k_{so} L^2$, $q^* = q/k_{so} L$, $q_u^* = q_u/k_{so} L$, $q_{cu}^* = q_{cu}/k_{so} L$, $\tau_u^* = \tau_u H/k_{so} L^2$, $\rho^* = \rho v^2/k_{so} L^2$, $I^* = EI/k_{so} L^4$, $c^* = cv/k_{so} L$, $P^* = P/k_{so} L^2$, $d\xi^* = d\xi/L$ and $\alpha = k_{co}/k_{so}$, P is the applied load and L is the half length of the beam considered. The above Eqs. (14) and (15), can be written in a non-dimensional form as

$$q^* = \frac{W}{U[1 + (W/q_u^*)]} - G^* \frac{d^2 W}{d\xi^{*2}} \quad (16a)$$

within the saturated soft soil region

$$q^* = \frac{\alpha W}{[1 + (W/q_{cu}^*)]} - G^* \frac{d^2 W}{d\xi^{*2}} \quad (16b)$$

within the stone column region

$$\frac{d^4 W}{d\xi^{*4}} + \left(\frac{\rho^*}{I^*}\right) \frac{d^2 W}{d\xi^{*2}} - \left(\frac{c^*}{I^*}\right) \frac{dW}{d\xi^*} + \frac{q^*}{I^*} = \frac{P^*(\xi^*)}{I^*} \quad (17)$$

where

$$G^* = \frac{G_o^*}{[1 + (G_o^* |dW/d\xi^*|)/\tau_u^*]^2} \quad (18)$$

Writing Eqs. (16) and (17) in a finite difference form, within a specified space domain and for an interior node, i , one gets

$$q_i^* = \frac{W_i}{U[1 + (W_i/q_u^*)]} - G_i^* \frac{W_{i-1} - 2W_i + W_{i+1}}{(\Delta\xi^*)^2} \quad (19a)$$

within the saturated soft soil region

$$q_i^* = \frac{\alpha W_i}{[1 + (W_i/q_{cu}^*)]} - G_i^* \frac{W_{i-1} - 2W_i + W_{i+1}}{(\Delta\xi^*)^2} \quad (19b)$$

within the stone column region

and

$$W_i = \frac{1}{6 - 2(\rho^*/I^*)(\Delta\xi^*)^2} \left[-\frac{q_i^*}{I^*} (\Delta\xi^*)^4 - W_{i-2} - W_{i-1} \left(-4 + \left(\frac{\rho^*}{I^*}\right) (\Delta\xi^*)^2 + \frac{c_i^*}{2I^*} (\Delta\xi^*)^3 \right) - W_{i+1} \left(-4 + \left(\frac{\rho^*}{I^*}\right) (\Delta\xi^*)^2 - \frac{c_i^*}{2I^*} (\Delta\xi^*)^3 \right) - W_{i+2} + \left(\frac{P_i^*}{I^*}\right) (\Delta\xi^*)^3 \right] \quad (20)$$

4.1. Boundary conditions

Boundary conditions have been considered at the ends of the beam. At both ends of the beam, the deflection and the slope of the deflected shape of the beam are zero. These boundary conditions have been written in a non-dimensional form and are as follows:

$$\text{At } \xi^* = -1 \text{ and } 1, \quad W = 0 \text{ and } \frac{dW}{d\xi^*} = 0$$

5. Convergence criterion and range in parametric values

Based on the formulation presented above, a computer program has been developed to obtain the response of an infinite beam–soil system using a finite difference scheme. The half length of the beam has been taken to be large enough so that the beam can be assumed to act as an infinite beam. The complete region of the problem ($-L \leq x \leq L$) has been considered for analysis. The total length of the beam ($2L$) was discretized with a finite difference method, and it was observed that the difference in responses corresponding to a finite difference mesh with 5001 nodes and one with 8001 nodes was less than 0.5%, and hence, the mesh with 5001 nodes was preferred for all parametric studies. The solution was obtained with a tolerance factor of 10^{-10} .

Due consideration has been given to the choice of realistic values for various parameters for the purpose of a parametric study. Although typical values for the different parameters relevant to the railway tracks considering the conditions in

Table 1
Range in values of various parameters considered for parametric study.

Parameter	Symbol	Range in values	Unit
Applied load	P	100–250	kN
Mass per unit length of beam	ρ	52	kg/m
Flexural rigidity of beam	EI	4.47×10^6 (Shahu et al., 2000)	N m ²
Initial modulus of subgrade reaction for soft foundation soil	k_{so}	15 (Das, 1999)	MN/m ²
Initial shear modulus of granular fill	G_o	652.4 (Desai and Abel, 1987)	kN/m ²
Ultimate bearing capacity of soft foundation soil	q_u	20–60	kN/m ²
Ultimate bearing capacity of stone column	q_{cu}	100–200	kN/m ²
Ultimate shear resistance of granular fill layer	τ_u	4–10	kN/m ²
Velocity of applied load	v	40–140	km/h
Thickness of granular fill layer	H	0.30	m
Diameter of stone columns	d	0.12–1.2	m
Spacing to diameter ratio	s/d	2–4	
Damping ratio	–	5–50%	–
Relative stiffness of stone columns	α	10–100	
Average degree of consolidation	U	40–100	%

Table 2
Range in values of non-dimensional parameters considered for parametric study.

Non-dimensional parameter	Expression	Range in values
P^*	$P/k_{so}L^2$	3×10^{-7} – 7.5×10^{-7}
ρ^*	$\rho v^2/k_{so}L^2$	1.5×10^{-7} – 1×10^{-5}
I^*	$EI/k_{so}L^4$	6×10^{-10}
G_o^*	$G_oH/k_{so}L^2$	6×10^{-7}
q_u^*	$q_u/k_{so}L$	2×10^{-4} – 6×10^{-4}
τ_u^*	$\tau_uH/k_{so}L^2$	1.2×10^{-6} – 3×10^{-6}

India have been adopted for the parametric study, the solution algorithm is general enough to take care of any set of realistic values of parameters. The parameters considered in this study (Table 1) have been non-dimensionalized for the purpose of analysis, and the ranges in values of such non-dimensionalized parameters are presented in Table 2.

While presenting the results, the coefficient of the characteristic wavelength of the unreinforced soil in the static case has been used for the normalization, which is expressed as (Mallik et al. 2006)

$$\lambda = (k_{so}/EI)^{1/4}$$

The distance along the rail, i.e., the x -axis, has been normalized by multiplying distance ξ^* (ahead and behind) from the load by λ . The y -axis has been normalized by dividing the response (deflection and bending moment) by the maximum value of these in the static case, i.e., when $v=0$. The amount of damping has been expressed as a percentage of the critical damping, which is $(2(k_s\rho)^{1/2})$.

6. Results and discussion

6.1. Validation

To validate the proposed model, the results from a degenerated case of the present study have been compared

to those of Mallik et al. (2006). They proposed a solution for the response of an infinite beam with a concentrated load moving at a constant velocity and resting on an unreinforced soil. For the undamped case, the response was obtained in a closed form; however, for the case where damping was present, the results were obtained numerically. The governing equation for the system considered was

$$\frac{d^4w}{d\xi^4} + \frac{\rho v^2}{EI} \frac{d^2w}{d\xi^2} - \frac{cv}{EI} \frac{dw}{d\xi} + \frac{kw}{EI} = \frac{P(\xi)}{EI} \quad (21)$$

where EI is the flexural rigidity of the infinite beam, ρ is the mass per unit length of the beam, c is the coefficient of viscous damping per unit length of the beam, v is the velocity of the moving load, ξ is a distance at any time $t > 0$ defined as $\xi = (x - vt)$ and $P(\xi)$ is the applied load intensity.

$w = e^{m\xi}$ was assumed as the solution for Eq. (21) and the four roots of the same for an underdamped case were obtained as

$$m_1 = -p + iq$$

$$m_2 = -p - iq$$

$$m_3 = p + ir \quad \text{and}$$

$$m_4 = p - ir$$

where p , q and r are real positive numbers. The solution for the differential equation (21), was given as

$$w_1(\xi) = e^{-p\xi} [A \cos q\xi + B \sin q\xi] \quad \text{for } \xi > 0 \quad \text{and} \quad (22a)$$

$$w_2(\xi) = e^{p\xi} [C \cos r\xi + D \sin r\xi] \quad \text{for } \xi < 0 \quad (22b)$$

The boundary conditions considered were

$$\left. \begin{aligned} w_1(0) &= w_2(0) \\ w_1'(0) &= w_2'(0) \\ w_1''(0) &= w_2''(0) \\ w_1'''(0) - w_2'''(0) &= P/EI \end{aligned} \right\} \quad (23)$$

where the prime denotes the differentiation with respect to ξ .

Using Eqs. (22) and (23), four equations were obtained for the evaluation of constants A , B , C and D , namely,

$$\left. \begin{aligned} A - C &= 0 \\ -2pA + Bq - Dr &= 0 \\ (r^2 - q^2)A - 2pqB - 2prD &= 0 \\ p(3q^2 - 2p^2 + 3r^2)A + q(3p^2 - q^2)B + r(r^2 - 3p^2)D &= P/EI \end{aligned} \right\} \quad (24)$$

The values of positive real numbers p , q and r and four constants A , B , C and D have been obtained here using Software “Mathematica” for the purpose of validation and comparison. A comparison has been made for the parameters as follows:

$\rho = 25 \text{ kg/m}$, $k = 40.78 \times 10^5 \text{ N/m}^2$, $EI = 1.75 \times 10^6 \text{ N m}^2$, $P = 93.36 \times 10^3 \text{ N/m}$, damping = 30% and velocity ratio = 0.50 (Mallik et al., 2006).

Such a comparison is presented in Fig. 3. A very good agreement of the responses in the form of the deflection of the infinite beam has been observed with the results of Mallik et al. (2006). This validates the proposed model, the methodology and the adopted solution technique.

6.2. Influence of ground-improvement techniques

The provision of stone columns is one of the most widely adopted ground-improvement techniques. Other ground-improvement techniques adopted in railways include the provision of a geosynthetic layer in between the ballast or a granular fill layer (Indraratna et al., 2006, 2007). Such a system has been considered by Maheshwari and Karuppasamy (2011) in which infinite beams on geosynthetic reinforced granular fill, overlying a relatively poor soil, have been analyzed under the action of moving loads, considering the nonlinear behavior and the viscous damping of the supporting poor soil subgrade system. In this case, the geosynthetic layer has been modeled as a rough elastic membrane. Tension in the geosynthetic gets mobilized due to the application of the load. A component of

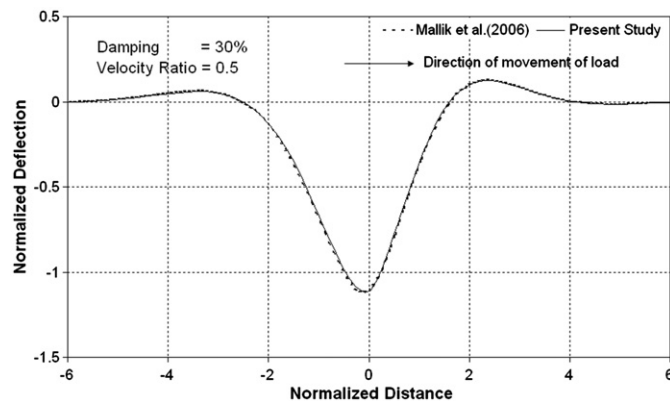


Fig. 3. Comparison of deflection profile of infinite beam with Mallik et al. (2006).

this mobilized tension resists some of the applied load, thereby transferring a lesser load to the underlying soft soil, and hence, a reduction in the settlement. However, by providing the stone columns, the reduction in settlement is achieved by accelerating the consolidation of the soft soil due to the shortened drainage path, by increasing the load-carrying capacity and/or by the decrease in settlement due to the inclusion of stronger granular material. Many times, stone columns and geosynthetics are used together for ground improvement. In such cases, either a geosynthetic layer is provided on top of the stone column-treated ground or geosynthetic-encased stone columns are provided.

Fig. 4 depicts a comparison of the deflection profiles of a beam for two ground-improvement techniques, viz., (i) the installation of stone columns and (ii) the provision of a geosynthetic layer, along with the deflection profile for the case of no ground improvement. Typical values for the parameters have been considered as $P^* = 1 \times 10^{-6}$, $\rho^* = 2.5 \times 10^{-7}$, $G_o^* = 6 \times 10^{-7}$, $I^* = 6 \times 10^{-10}$, $q_u^* = 1.8 \times 10^{-5}$, $q_{cu}^* = 2.5 \times 10^{-6}$, $\tau_u^* = 5.4 \times 10^{-9}$, $d/L = 0.004$, $s/d = 3$, $\alpha = 25$ and damping = 30%. It can be observed that the maximum normalized deflection decreases from 0.001477 to 0.001356 due to inclusion of a geosynthetic layer. However, with the provision of stone columns, the normalized deflection has been found to decrease drastically from 0.001477 to 0.000236. This depicts the advantage of employing stone columns in reducing the settlement over other methods of ground improvement usually adopted in the case of railways. The relative influence of other parameters, like the ratio, s/d , traffic load velocities, the strength of the soft foundation, etc. can be observed in the subsequent paragraphs related to the respective parametric studies.

6.3. Influence of magnitude of applied load (P^*)

Figs. 5 and 6 show the influence of an applied load on the deflection and the bending moment of an infinite beam, respectively, for parameters $\rho^* = 2.5 \times 10^{-7}$, $G_o^* = 6 \times 10^{-7}$,

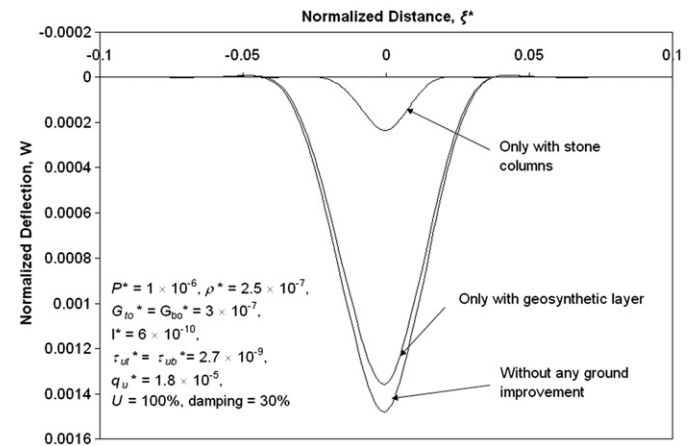


Fig. 4. Influence of ground improvement techniques on deflection of beam.

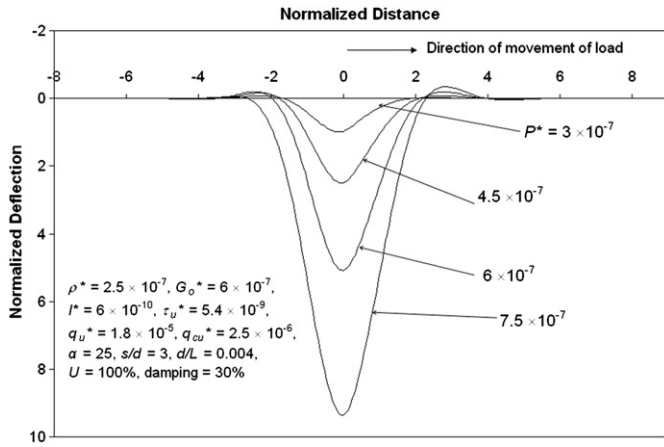


Fig. 5. Deflection profiles of beam: influence of applied load.

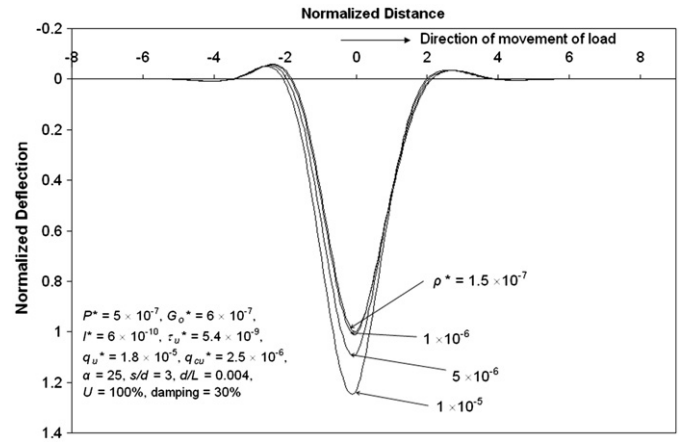


Fig. 7. Deflection profiles of beam: influence of velocity of load.

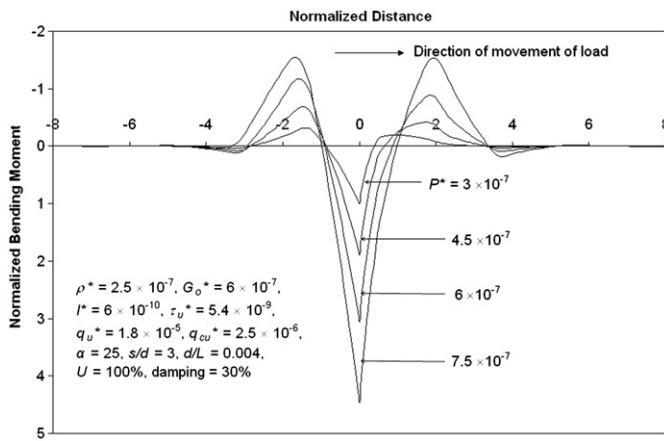


Fig. 6. Bending moment variation: influence of applied load.

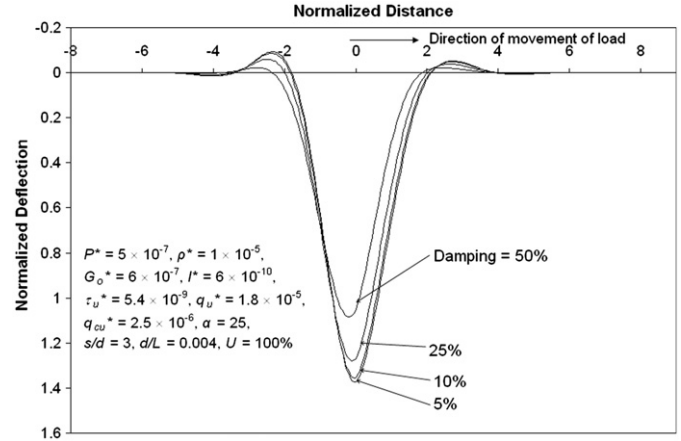


Fig. 8. Deflection profiles of beam: influence of damping.

$I^* = 6 \times 10^{-10}$, $q_u^* = 1.8 \times 10^{-5}$, $q_{cu}^* = 2.5 \times 10^{-6}$, $\tau_u^* = 5.4 \times 10^{-9}$, $d/L = 0.004$, $s/d = 3$, $\alpha = 25$, damping = 30% and $U = 100\%$. Normalized load P^* has been varied from 7.5×10^{-7} to 3×10^{-7} and the corresponding reduction in maximum normalized deflection has been found to be 89.4% (Fig. 5). Fig. 6 shows the variation in normalized bending moment along the length of the beam for different values of normalized applied load. It can be observed that positive as well as negative bending moments decrease with a reduction in the applied load. The reduction in maximum positive normalized bending moment has been found to be about 77% as the applied load is reduced from 7.5×10^{-7} to 3×10^{-7} (Fig. 6).

6.4. Influence of velocity of applied load (ρ^*)

Fig. 7 presents the influence of the velocity of the applied load on the deflection of the infinite beam for typical values of non-dimensional input parameters, as mentioned in the figure. It can be observed that as the velocity is increased from 1.5×10^{-7} to 1×10^{-5} , the normalized deflection also increases and this increase in maximum normalized deflection has been found to be 23.4%.

Furthermore, it is clear that at lower velocities, the deflection of the beam is not affected significantly by any change in the velocity of the applied load. The influence of velocity on the bending moment of the beam has not been found to be significant. However, an increase in velocity results in a marginal increase in positive as well as negative normalized bending moments in the beam.

6.5. Influence of damping

Figs. 8 and 9 depict the effect of damping on the response of the beam–soil system under consideration with respect to deflection and the bending moment in the beam, respectively. It was observed that at lower values of velocity (Fig. 7), the damping has a negligible effect on the response, and therefore, this parametric study has been conducted at the high value of velocity of 1×10^{-5} . The values of other input parameters considered are $P^* = 5 \times 10^{-7}$, $G_o^* = 6 \times 10^{-7}$, $I^* = 6 \times 10^{-10}$, $q_u^* = 1.8 \times 10^{-5}$, $q_{cu}^* = 2.5 \times 10^{-6}$, $\tau_u^* = 5.4 \times 10^{-9}$, $d/L = 0.004$, $s/d = 3$, $\alpha = 25$ and $U = 100\%$. The damping ratio has been varied from 5% to 50% and, as expected, the normalized deflection has been found to decrease with an increase in damping (Fig. 8).

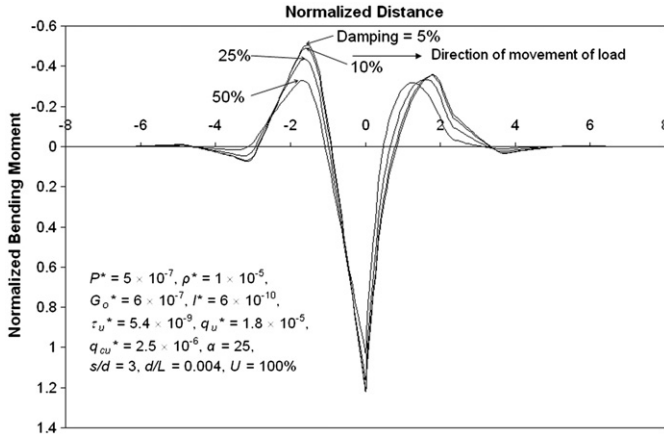


Fig. 9. Bending moment variation: influence of damping.

The maximum deflection has been found to decrease by about 31% as the damping ratio is increased from 5% to 50%. In addition, the location of the occurrence of maximum deflection has been found to shift behind the point of the application of the load as the damping increases. The variation in normalized bending moment along the length of the beam, for various values of damping ratio, is shown in Fig. 9 and the maximum positive bending moment has been found to decrease by about 19% as the damping ratio is increased from 5% to 50%.

6.6. Influence of diameter of stone columns (d)

The effect of the diameter of the stone columns on the response of the system has been presented in a non-dimensional form and, for this purpose, the diameter of a stone column (d) has been non-dimensionalized with a half length of the beam (L). The normalized diameter has been varied from 0.0008 to 0.008 and other input parameters have been considered as $P^* = 5 \times 10^{-7}$, $\rho^* = 2.5 \times 10^{-7}$, $G_o^* = 6 \times 10^{-7}$, $I^* = 6 \times 10^{-10}$, $q_u^* = 1.8 \times 10^{-5}$, $q_{cu}^* = 2.5 \times 10^{-6}$, $\tau_u^* = 5.4 \times 10^{-9}$, $s/d = 3$, $\alpha = 25$, damping = 30% and $U = 100\%$. The influence of the normalized diameter (d/L) on deflection and the bending moment in the infinite beam is presented in Figs. 10 and 11, respectively, keeping the ratio s/d constant. It has been observed that for a constant value of spacing to diameter ratio, the deflection of the footing increases as the diameter of the stone columns is increased and beyond a certain value, any further increase in diameter reduces the deflection of the beam (Fig. 10). The maximum normalized deflection of the beam has been found to increase by 65.6% as the normalized diameter of the stone columns is increased from 0.0008 to 0.006. However, a reduction of about 28% has been observed as the normalized diameter is increased from 0.006 to 0.008. As the diameter is increased, the number of stone columns beneath the footing decreases, which causes an increase in the deflection. However, an increase in the stone column diameter also results in the

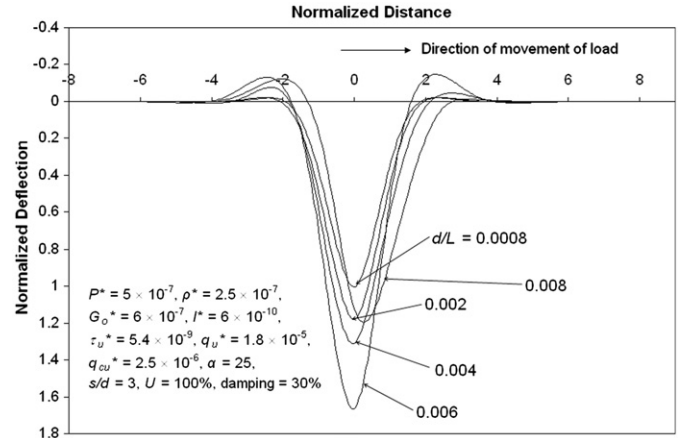


Fig. 10. Deflection profiles of beam: influence of diameter of stone columns.

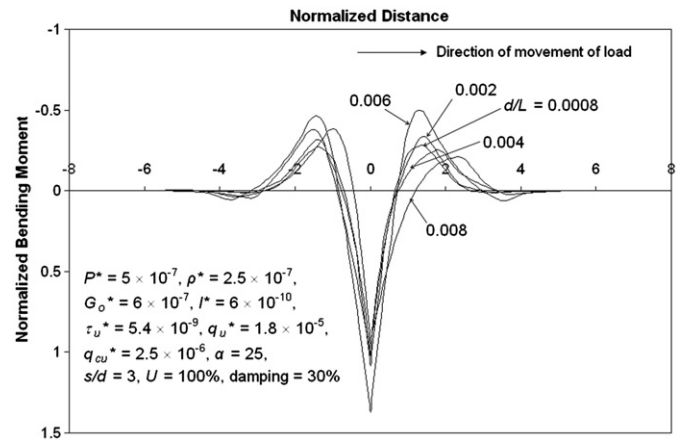


Fig. 11. Bending moment variation: influence of diameter of stone columns.

replacement of a larger portion of soft soil by granular material, thereby reducing the settlement. The influence of a reduction in the number of stone columns has been found to be dominating as compared to the influence of replacing the soft soil by better granular material for d/L values lying between 0.0008 and 0.006. For $d/L = 0.008$, the effect of the replacement of a larger portion of soft soil by granular material has been observed to be more pronounced. This explains the influence of the diameter of the stone columns on the deflection of the infinite beam under moving loads.

6.7. Influence of spacing to diameter ratio (s/d)

The effect of the spacing to diameter ratio of the stone columns on deflection and the bending moment in the infinite beam is presented in Figs. 12 and 13, respectively, for typical values of input parameters, as mentioned in figures. It can be observed that normalized maximum deflection increases with an increase in the s/d ratio. For a particular value of diameter of stone columns, an

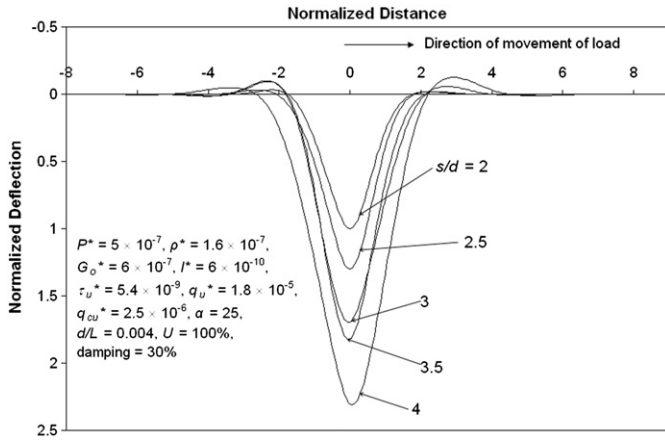


Fig. 12. Deflection profiles of beam: influence of spacing to diameter ratio.

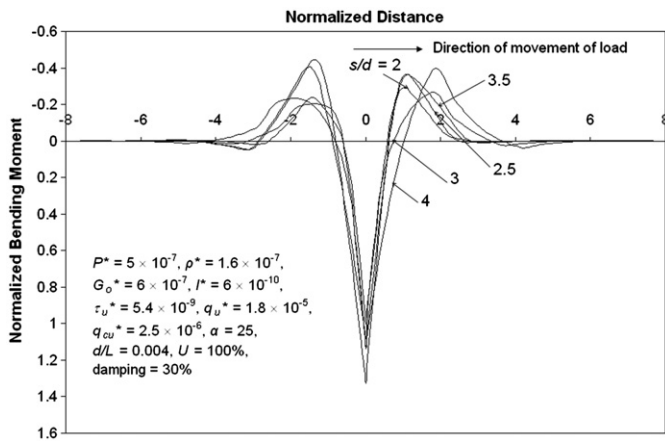


Fig. 13. Bending moment variation: influence of spacing to diameter ratio.

increase in spacing reduces the number of stone columns in the treated area and results in an increase in the normalized deflection. The maximum normalized deflection has been found to reduce by 56.5% as the s/d ratio is reduced from 4 to 2 (Fig. 12). Furthermore, it can also be observed that there is only a marginal difference in the deflection profiles of the beam for s/d ratios of 3 and 3.5. Fig. 13 shows that positive as well as negative bending moments in the beam are significantly affected by any variation in the s/d ratio.

6.8. Influence of ultimate bearing resistance of soft foundation soil (q_u)

Fig. 14 depicts the influence of the ultimate bearing resistance of soft foundation soil on the deflection of the infinite beam. Non-dimensional parameter q_u^* is varied from 8×10^{-6} to 3×10^{-5} , while keeping all other parameters constant, as mentioned in the figure. As was expected, the corresponding reduction in the maximum normalized deflection has been found to be about 27%. The influence of q_u^* on the bending moment in the beam has been found to be negligible.

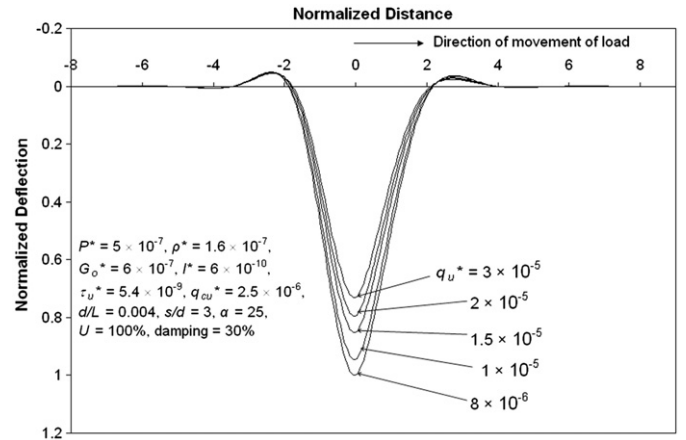


Fig. 14. Deflection profiles of beam: influence of ultimate bearing resistance of soft foundation soil.

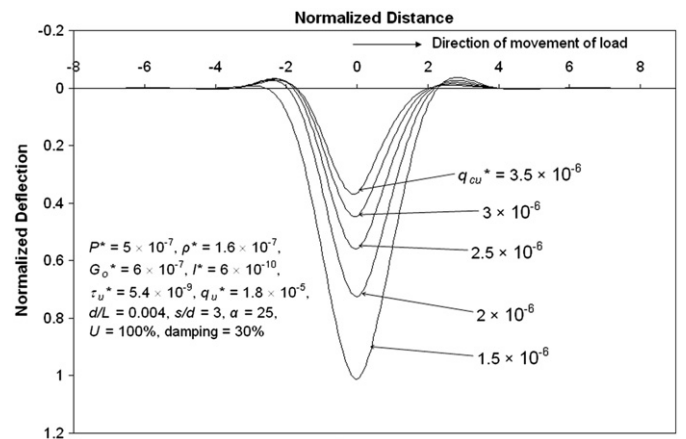


Fig. 15. Deflection profiles of beam: influence of ultimate bearing resistance of stone columns.

6.9. Influence of ultimate bearing resistance of stone columns (q_{cu})

The effect of the ultimate bearing resistance of the stone columns, q_{cu} , on the deflection of the infinite beam is presented in Fig. 15. A reduction of about 64% in maximum normalized deflection has been observed; this corresponds to an increase in q_{cu}^* from 1.5×10^{-6} to 3.5×10^{-6} . The bending moment in the beam has also been found to be significantly affected by any variation in parameter q_{cu}^* ; this has been presented in Fig. 16. A reduction in the maximum positive bending moment of about 37% has been observed; this corresponds to an increase in q_{cu}^* from 1.5×10^{-6} to 3.5×10^{-6} .

6.10. Influence of ultimate shear resistance of granular fill (τ_u)

The effect of the ultimate shear resistance of granular fill on the deflection of a beam is presented in Fig. 17 for typical values of input parameters, as mentioned in the figure. Parameter τ_u^* has been varied from 3.6×10^{-9} to 9.0×10^{-6} ,

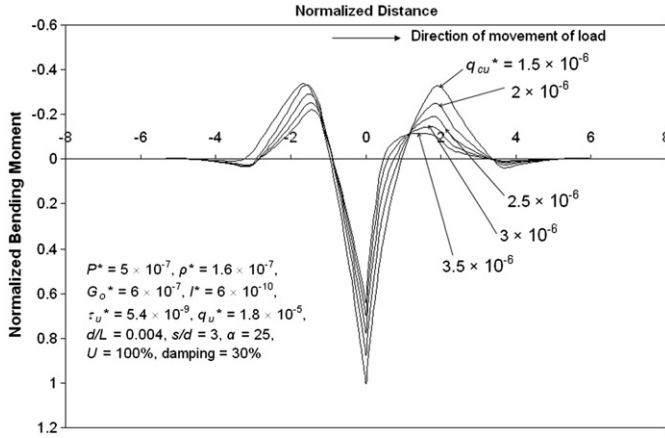


Fig. 16. Bending moment variation: influence of ultimate bearing resistance of stone columns.

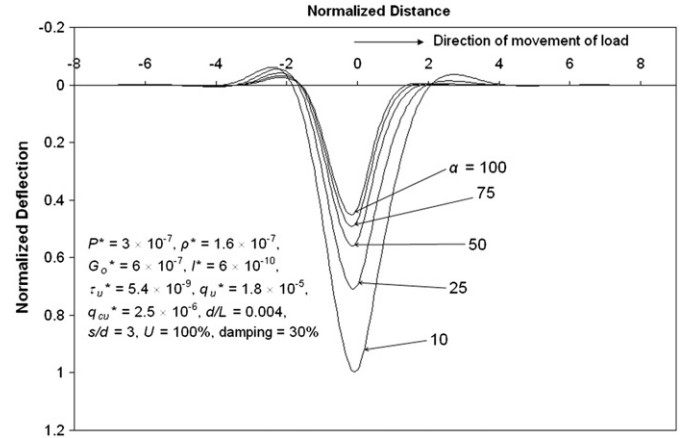


Fig. 18. Deflection profiles of beam: influence of relative stiffness of stone columns with surrounding soil.

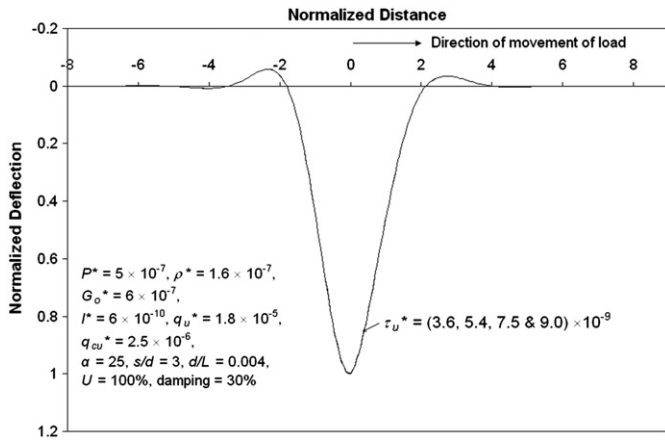


Fig. 17. Deflection profiles of beam: influence of ultimate shear resistance of granular fill.

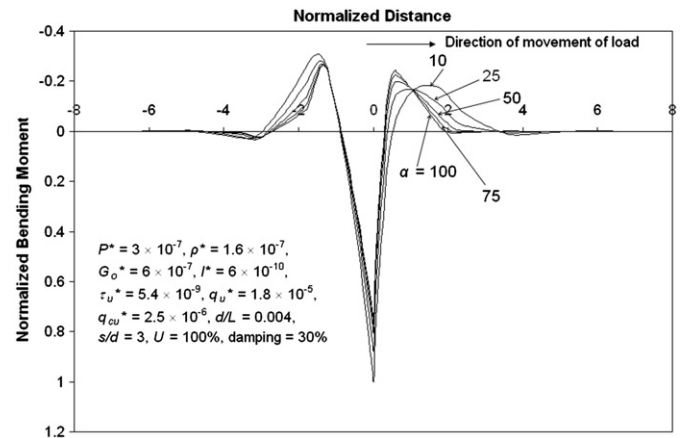


Fig. 19. Bending moment variation: influence of relative stiffness of stone columns with surrounding soil.

and it has not been found to influence the response of the soil–foundation system under consideration.

6.11. Influence of relative stiffness of stone columns with surrounding soil ($\alpha = k_{co}/k_{so}$)

Fig. 18 depicts the variation in the deflection of the infinite beam for various values of parameter $\alpha (=k_{co}/k_{so})$, varying from 10 to 100. Other input parameters have been mentioned in the figure. The maximum normalized deflection has been found to decrease by about 57% as parameter α is increased from 10 to 100. An increase in parameter α indicates the larger stiffness of the natural soil, and therefore, a reduction in the deflection has been observed for larger values of parameter α . Fig. 19 shows the influence of parameter α on the bending moment in the infinite beam. The negative bending moment has been found to be more sensitive towards any variation in parameter α than the positive bending moment.

6.12. Influence of average degree of consolidation (U)

The influence of the average degree of consolidation, U , on the deflection of the beam is presented in Fig. 20 for

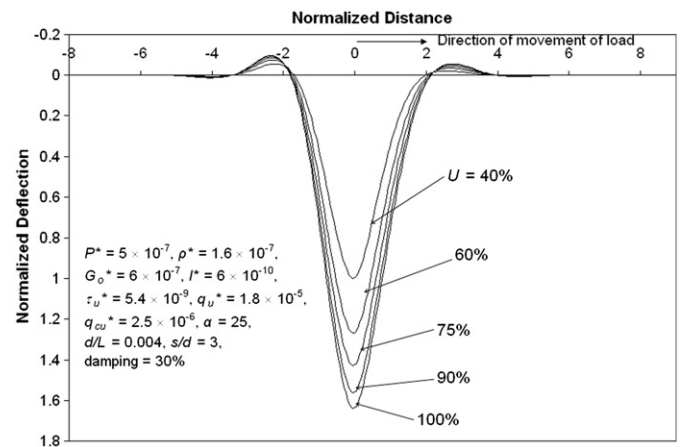


Fig. 20. Deflection profiles of beam: influence of average degree of consolidation.

typical values of input parameters as $P^* = 5 \times 10^{-7}$, $\rho^* = 1.6 \times 10^{-7}$, $G_o^* = 6 \times 10^{-7}$, $I^* = 6 \times 10^{-10}$, $q_u^* = 1.8 \times 10^{-5}$, $q_{cu}^* = 2.5 \times 10^{-6}$, $\tau_u^* = 5.4 \times 10^{-9}$, $d/L = 0.004$, $s/d = 3$,

$\alpha=25$ and damping=30%. An increase of about 64% in the maximum normalized deflection has been observed as the average degree of consolidation increases from 40% to 100%.

7. Summary and concluding remarks

In the present study, a simplified model has been proposed for the nonlinear analysis of an infinite beam, subjected to a concentrated load moving at a constant velocity and resting on a granular bed on top of a stone column-treated soft ground. The nonlinear behavior of the soft soil, the stone columns and the granular bed has been considered by hyperbolic constitutive relationships. Different parameters relevant to the system under consideration have been chosen for the performance of a detailed parametric study. The response of the system has been presented in the form of deflection and the bending moment in the beam. All the results of the parametric study have been presented in a non-dimensional form which can be used readily. Based on this, the following generalized conclusions can be drawn:

- i. Any variation in applied load has been found to significantly influence the response of the infinite beam with respect to its deflection and bending moment. A reduction in the maximum normalized deflection has been found to be about 90% as the normalized load, P^* , was varied from 7.5×10^{-7} to 3×10^{-7} .
- ii. The deflection of the infinite beam is not affected at lower values of velocity for the applied load. However, at larger values of velocity, the deflection has been found to be sensitive towards any changes in velocity.
- iii. The influence of damping on the response of the system has been found to be negligible at lower values of velocity. However, at higher velocities, as expected, the deflection of the beam has been found to decrease with an increase in the damping; this decrease has been found to be in the order of 30% for typical values of the input parameters considered.
- iv. For a particular spacing to diameter of the stone columns ratio, the deflection of the beam has been found to increase with an increase (65.6%) in diameter of the stone column (d/L varying from 0.0008 to 0.006). Further increases in the diameter reduce the deflection of the beam (28%), i.e., from 0.006 to 0.008.
- v. A variation in the s/d ratio suggested that the deflection of the foundation beam decreases with a decrease in the s/d ratio. For the range in values of the input parameters considered here, an optimum value for the s/d ratio can be considered as 3–3.5.
- vi. The deflection of the foundation beam has been found to decrease with an increase in q_u . This reduction has been found to be about 27%, which corresponds to a reduction in q_u^* from 8×10^{-6} to 3×10^{-5} . The influence of q_u^* on the bending moment in the beam has been found to be negligible.
- vii. The ultimate bearing capacity of stone columns, q_{cu} , has been found to significantly influence the response of the soil–foundation system. A reduction of 64% in maximum normalized deflection in the footing has been observed; this corresponds to an increase in q_{cu}^* from 1.5×10^{-6} to 3.5×10^{-6} . The corresponding reduction in the maximum positive normalized bending moment has been observed to be about 37%.
- viii. The relative stiffness of the stone columns with the surrounding soil has been found to greatly influence the response of the soil–foundation system. The maximum normalized deflection has been found to decrease by about 57% as parameter α is increased from 10 to 100. The negative bending moment has been found to be more sensitive towards any variation in parameter α than the positive bending moment.
- ix. An increase of about 64% in the maximum normalized deflection has been observed as the average degree of consolidation is increased from 40% to 100%.
- x. The response of the system has been found to be independent of any variation in ultimate shear resistance of the granular fill layer.

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